MATH 580: Review of Matrices

Fall 2004 AB, Penn State

Given B, a $m \times n$ (rows by columns) matrix:

1. $B^T B$ is square and symmetric
2. if B has linearly independent columns (i.e. B has rank n, assuming $n < m$) then $B^T B$ is symmetric positive definite, which means that $(B^T B)^{-1}$ exists.

For a square matrix, A $n \times n$, any of the following is necessary and sufficient for A to be nonsingular: (thus any one implies the others)

1. the columns a of A span $\mathbb{R}^n$, i.e. $a \in R(A)$
2. the columns are linearly independent, which means that $Ax = 0$ has only the solution $x = 0$, i.e. the nullspace $N(A) = \{0\}$
3. the rows of A span $\mathbb{R}^n$
4. the rows are linearly independent
5. $\det A \neq 0$
6. the inverse A^{-1} exists, such that $A^{-1}A = AA^{-1} = \mathbb{I}$
7. For $Ax = \lambda x$, $\lambda \neq 0$ (there are no zero eigenvalues of A)
8. $A^T A$ is symmetric positive definite