

**MATH 580: Problem Set 1**  
**due Tuesday, September 14, 2004**

1. Are  $y^2$  and  $y^4$  linearly independent? Why or why not?
2. Keener 1.1.3a (p.49)
3. Keener 1.1.5 (p.49)
4. Keener 1.1.8 (p.49)
5. Why isn't  $\|x\| \equiv \langle x, x \rangle$  a valid induced norm?
6. Let  $f_1, f_2, f_3, \dots$  be a set of linearly independent functions in the space  $L^2[a, b]$ , which means functions square-integrable on the interval  $[a, b]$ , i.e.

$$\int_a^b [f(x)]^2 dx < \infty$$

Using the Gram-Schmidt orthogonalization process, define a sequence of *orthogonal* functions  $g_i$  in terms of the  $f_i$ .

7. Using the generalized definition of the angle between two elements of an inner product space,
  - (a) show that the angle between  $x$  and  $x^2$  is about  $14.5^\circ$ , where the inner product

$$\langle f, g \rangle \equiv \int_0^2 f(x)g(x) dx;$$

- (b) what would the angle between the same two functions be for the inner product

$$\langle f, g \rangle \equiv \int_1^6 f(x)g(x) dx$$