MATH 406: Problem Set 13

due Wednesday, April 27, 2005

1. (a) Without using any facts about electromagnetism (except that the electrostatic potential is harmonic), guess the functional dependence of the potential \( \phi \) for a purely radial electric field, which is due to an isolated electric charge (such as an electron). Define your reasoning clearly, and specify the associated analytic function in \( \mathbb{C} \).

(b) Using superposition, find the total electric potential due to this charge in a spatially constant field \( E_0 \).

2. (a) Show that the Zhukovsky mapping \( w = z + \left( A^2/z \right) \) has a relation with the Milne-Thompson Circle Theorem for a circle of radius \( R = A \).

(b) (based on Fisher 4.2.15) Show that the Zhukovsky map transforms flow past a circle with \( R < A \) to flow past an ellipse with axes \( a \) and \( b < a \) (see Fisher p.282).

(c) Describe what happens when \( A \to R \). Why is this different from your answer to part (a)? HINT: it is.

3. Find the Fourier transform \( \hat{u}(\omega) \) for the following functions of time \( t \):

   a) \( u(t) = \frac{5}{1 + t^2} \)

   b) \( u(t) = \frac{1}{20 + 8t + t^2} \) (F 5.1.5)