There are 10 problems in this exam (6 multiple choice and 4 partial credit problems).

Circle exactly one answer for the multiple choice problems.

Present your work clearly for the partial credit problems. **No credit will be given for unsupported answers.**

No calculators, books, or notes is permitted in this exam.

Box your final answers whenever possible.

Turn off your cell phone before the exam starts.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 10 PROBLEMS ON 8 PAGES (INCLUDING THIS ONE).

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1. (5 pts) Which one of the following combinations is the center and radius of the sphere 
\[ 4x^2 + 4y^2 + 4z^2 - 8x + 12y - 16z + 13 = 0 \]?

a) \((1, -\frac{3}{2}, 2)\) and 4  
b) \((1, \frac{3}{2}, 2)\) and 2  
c) \((1, -\frac{3}{2}, 2)\) and 2  
d) \((1, \frac{3}{2}, 2)\) and 4

2. (5 pts) Given \(\mathbf{a} = \langle 1, 0, 1 \rangle\), which one of the following corresponds to the vector in the \textbf{opposite} direction of \(\mathbf{a}\) with a magnitude of \(2\sqrt{2}\)?

a) \(\langle -2\sqrt{2}, 0, -2\sqrt{2} \rangle\)  
b) \(\langle 0, -2\sqrt{2}, 0 \rangle\)  
c) \(\langle -2, 0, -2 \rangle\)  
d) \(\langle \sqrt{2}, 0, \sqrt{2} \rangle\)
3. (5 pts) Which one of the following equations corresponds to the plane through the point
$P(1, 2, -1)$ with normal direction $\mathbf{n} = \langle 2, -1, 3 \rangle$?

a) $x = 1 + 2t, \ y = 2 - t, \ z = -1 + 3t$

b) $x = -1 + 2t, \ y = 3 - t, \ z = -4 + 3t$

c) $2x - y + 3z + 3 = 0$

d) $2x - y + 3z = 3$

4. (5 pts) Which one of the following equations could be the quadric surface in Figure 1?

![Figure 1: Some quadric surface in rectangular coordinates](image)

a) $z = x^2 + \frac{y^2}{2}$

b) $x^2 + \frac{y^2}{2} + z^2 = 1$

b) $z^2 = x^2 + \frac{y^2}{2}$

d) $z = x^2 - \frac{y^2}{2}$
5. (5 pts) Find the spherical coordinates of the point with cylindrical coordinates \((2, \frac{\pi}{3}, -2)\).

   a) \((2, \frac{\pi}{3}, -\frac{\pi}{4})\)
   
   b) \((2, \frac{\pi}{3}, \frac{\pi}{4})\)
   
   c) \((2\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{4})\)
   
   d) \((2\sqrt{3}, \frac{\pi}{3}, \frac{\pi}{4})\)

6. (5 pts) Find symmetric equations for the tangent line to the curve given by \(\mathbf{r}(t) = <\ln t, 2\sqrt{t}, t^2>\) at the point \((0, 2, 1)\).

   a) \(x - 2 = y = \frac{z - 1}{2}\)
   
   b) \(x = \frac{y - 2}{2} = z - 1\)
   
   c) \(x = y - 2 = \frac{z - 1}{2}\)
   
   d) \(x = y - 2 = z - 1\)
7. (18 pts) Let \( \mathbf{a} = \langle 2, 2, -1 \rangle \) and \( \mathbf{b} = \langle 4, 1, 1 \rangle \).

a. Find the angle \( \theta \) between these two vectors \( \mathbf{a} \) and \( \mathbf{b} \).

b. Find an equation of the line through the point \( (1, 2, 3) \) and perpendicular to both \( \mathbf{a} \) and \( \mathbf{b} \).

c. Let \( \mathbf{c} = \langle 1, 1, 1 \rangle \). Find the volume of the parallelepiped determined by \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{c} \).
8. (18 pts) Consider two planes

\[ P_1 : 2x - y + 2z = 1 \]
\[ P_2 : x + 2y + z = 1 \]

a. Let \( \theta \) be the angle between these planes. Find \( \cos \theta \).

b. Find symmetric equations for the line of intersection of these two planes.

c. Find the distance from the point \((1, -1, 0)\) to the plane \( P_1 \).
9. (24 pts) Let $C$ be a space curve given by $r(t) = \left< 2\sin(t), 2\cos(t), \frac{t}{2} \right>$.

a. Find $\lim_{t \to 0} r(t)$.

b. Find the arc length of $C$ from $t = 0$ to $t = 1$.

c. Find the curvature of $C$ at the point $(2, 0, \pi/4)$.

d. Find an equation of the normal plane at the point $(2, 0, \pi/4)$. 

10. (10 pts) A particle moves with position function \( \mathbf{r}(t) = \langle e^t, t \cos(\frac{\pi t}{2}), e^{-t} \rangle \).

(a) Find the velocity of the moving particle at time \( t = 1 \).

(b) Find the acceleration of the moving particle at time \( t = 1 \).