

MATH 231

NAME \_\_\_\_\_

MIDTERM EXAM

STUDENT NUMBER \_\_\_\_\_

October 25, 2007

INSTRUCTOR \_\_\_\_\_

SECTION NUMBER \_\_\_\_\_

- There are 10 problems in this exam (6 multiple choice and 4 partial credit problems).
- Circle exactly one answer for the multiple choice problems.
- Present your work clearly for the partial credit problems. **No credit will be given for unsupported answers.**
- **No calculators, books, or notes is permitted in this exam.**
- Box your final answers whenever possible.
- Turn off your cell phone before the exam starts.

**CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 10 PROBLEMS ON 8 PAGES (INCLUDING THIS ONE).**

Question	SCORE
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
<b>TOTAL</b>	

1. (5 pts) Which one of the following combinations is the center and radius of the sphere  $4x^2 + 4y^2 + 4z^2 - 8x + 12y - 16z + 13 = 0$ ?

- a)  $(1, -\frac{3}{2}, 2)$  and 4
- b)  $(1, \frac{3}{2}, 2)$  and 2
- c)  $(1, -\frac{3}{2}, 2)$  and 2
- d)  $(1, \frac{3}{2}, 2)$  and 4

2. (5 pts) Given  $\mathbf{a} = \langle 1, 0, 1 \rangle$ , which one of the following corresponds to the vector in the **opposite** direction of  $\mathbf{a}$  with a magnitude of  $2\sqrt{2}$ ?

- a)  $\langle -2\sqrt{2}, 0, -2\sqrt{2} \rangle$
- b)  $\langle 0, -2\sqrt{2}, 0 \rangle$
- c)  $\langle -2, 0, -2 \rangle$
- d)  $\langle \sqrt{2}, 0, \sqrt{2} \rangle$

3. (5 pts) Which one of the following equations corresponds to the plane through the point  $P(1, 2, -1)$  with normal direction  $\mathbf{n} = \langle 2, -1, 3 \rangle$ ?

- a)  $x = 1 + 2t, y = 2 - t, z = -1 + 3t$
- b)  $x = -1 + 2t, y = 3 - t, z = -4 + 3t$
- c)  $2x - y + 3z + 3 = 0$
- d)  $2x - y + 3z = 3$

4. (5 pts) Which one of the following equations could be the quadric surface in Figure 1?

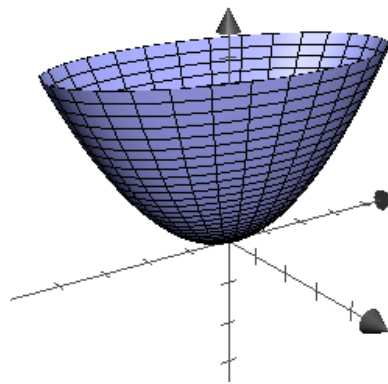


Figure 1: Some quadric surface in rectangular coordinates

- a)  $z = x^2 + \frac{y^2}{2}$
- b)  $x^2 + \frac{y^2}{2} + z^2 = 1$
- c)  $z^2 = x^2 + \frac{y^2}{2}$
- d)  $z = x^2 - \frac{y^2}{2}$

5. (5 pts) Find the spherical coordinates of the point with cylindrical coordinates  $(2, \frac{\pi}{3}, -2)$ .

a)  $(2, \frac{\pi}{3}, -\frac{\pi}{4})$

b)  $(2, \frac{\pi}{3}, \frac{\pi}{4})$

c)  $(2\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{4})$

d)  $(2\sqrt{3}, \frac{\pi}{3}, \frac{\pi}{4})$

6. (5 pts) Find symmetric equations for the tangent line to the curve given by  $\mathbf{r}(t) = \langle \ln t, 2\sqrt{t}, t^2 \rangle$  at the point  $(0, 2, 1)$ .

a)  $x - 2 = y = \frac{z-1}{2}$

b)  $x = \frac{y-2}{2} = z - 1$

c)  $x = y - 2 = \frac{z-1}{2}$

d)  $x = y - 2 = z - 1$

7. (18 pts) Let  $\mathbf{a} = \langle 2, 2, -1 \rangle$  and  $\mathbf{b} = \langle 4, 1, 1 \rangle$ .
- Find the angle  $\theta$  between these two vectors  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Find an equation of the line through the point  $(1, 2, 3)$  and perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Let  $\mathbf{c} = \langle 1, 1, 1 \rangle$ . Find the volume of the parallelepiped determined by  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$ .

8. (18 pts) Consider two planes

$$\mathcal{P}_1 : 2x - y + 2z = 1$$

$$\mathcal{P}_2 : x + 2y + z = 1$$

- a. Let  $\theta$  be the angle between these planes. Find  $\cos \theta$ .
- b. Find symmetric equations for the line of intersection of these two planes.
- c. Find the distance from the point  $(1, -1, 0)$  to the plane  $\mathcal{P}_1$ .

9. (24 pts) Let  $C$  be a space curve given by  $\mathbf{r}(t) = \left\langle 2 \sin(t), 2 \cos(t), \frac{t}{2} \right\rangle$ .
- Find  $\lim_{t \rightarrow 0} \mathbf{r}(t)$ .
  - Find the arc length of  $C$  from  $t = 0$  to  $t = 1$ .
  - Find the curvature of  $C$  at the point  $(2, 0, \pi/4)$ .
  - Find an equation of the normal plane at the point  $(2, 0, \pi/4)$ .

10. (10 pts) A particle moves with position function  $\mathbf{r}(t) = \langle e^t, t \cos(\frac{\pi t}{2}), e^{-t} \rangle$ .
- Find the velocity of the moving particle at time  $t = 1$ .
  - Find the acceleration of the moving particle at time  $t = 1$ .