• There are 16 problems in this exam (8 multiple choice and 8 partial credit problems).
• Circle exactly one answer for the multiple choice problems.
• Present your work clearly for the partial credit problems. **No credit will be given for unsupported answers.**
• No calculators, books, or notes is permitted in this exam.
• Box your final answers whenever possible.
• Turn off your cell phone before the exam starts.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 16 PROBLEMS ON 13 PAGES (INCLUDING THIS ONE).

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1. (5 pts) Find the angle between the following two vectors \( \mathbf{u} \) and \( \mathbf{v} \).

\[ \mathbf{u} = \langle 2, -2, 1 \rangle, \quad \mathbf{v} = \langle 1, -4, -1 \rangle \]

a) 0
b) \( \frac{\pi}{6} \)
c) \( \frac{\pi}{4} \)
d) \( \frac{\pi}{3} \)

2. (5 pts) The spherical coordinates of a point are \((\sqrt{3}, \pi/3, \pi/6)\). Find the rectangular coordinates of the point.

a) \( \left( \frac{\sqrt{3}}{4}, \frac{3}{4}, \frac{\sqrt{3}}{2} \right) \)
b) \( \left( \frac{3}{4}, \frac{3}{4}, \frac{3}{2} \right) \)
c) \( \left( \frac{3}{4}, \frac{3\sqrt{3}}{4}, \frac{3}{2} \right) \)
d) \( \left( \frac{\sqrt{3}}{4}, \frac{3}{4}, \frac{3}{2} \right) \)
3. (5 pts) Find symmetric equations for the tangent line to the curve
\[ \mathbf{r}(t) = (t + 1)\mathbf{i} + (t^2 + 1)\mathbf{j} + e^t\mathbf{k} \] at (1, 1, 1)

a) \( x - 1 = y - 1 = z - 1 \)
b) \( x - 1 = z - 1, y = 1 \)
c) \( x = z, y = 0 \)
d) \( x = y = z \)

4. (5 pts) A particle moves with position function
\[ \mathbf{r}(t) = < \cos t, e^{-t}, \sin t >. \]
Find the acceleration of the particle.

a) \( < \cos t, e^{-t}, \sin t > \)
b) \( < - \sin t, -e^{-t}, \cos t > \)
c) \( < - \sin t, e^{-t}, -\cos t > \)
d) \( < - \cos t, e^{-t}, -\sin t > \)
5. (5 pts) Find the domain of the following function.

\[ f(x, y) = \sqrt{4 - x^2 - y^2} \]

a) \( \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \geq 4\} \)

b) \( \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 > 4\} \)

c) \( \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 4\} \)

d) \( \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 < 4\} \)

6. (5 pts) Find the following limit.

\[ \lim_{{(x,y) \to (0,0)}} \frac{x}{x + y^2} \]

a) 0

b) 1

c) \( \infty \)

d) Not Exist
7. (5 pts) Let \( f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \). Find \( \frac{\partial f}{\partial z} \).

a) \( \frac{z}{\sqrt{x^2 + y^2 + z^2}} \)

b) \( \frac{x + y + z}{\sqrt{x^2 + y^2 + z^2}} \)

c) \( \frac{xyz}{\sqrt{x^2 + y^2 + z^2}} \)

d) 1

8. (5 pts) Find the gradient vector \( \nabla f \) of the following function.

\[ f(x, y) = 2\sqrt{x} - y^3 \] at \( (1, 3) \)

a) \( < 1, -27 > \)

b) \( < 2, -27 > \)

c) -26

d) 26
9. (15 pts) Consider the curve given by
\[ \mathbf{r}(t) = \langle \frac{1}{3} t^3, \frac{1}{2} t^2, t \rangle \]

a. Find the unit tangent vector.
b. Find \( \mathbf{r}'(t) \times \mathbf{r}''(t) \).
c. Find the curvature at the point \( (\frac{1}{3}, \frac{1}{2}, 1) \).
10. (10 pts) Let $C$ be a space curve given by

$$r(t) = < e^t, \cos t, \sin t >$$

Find the arc length of $C$ from $t = 0$ to $t = 1$. 
11. (10 pts) If \( xz^2 + y^2 z = e^{xyz} \), find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \).
12. (15 pts) Let $f(x, y) = x \cos(\pi y)$.
   
a. Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
   
b. Write the linearization $L(x, y)$ of $f(x, y)$ with respect to the point $(0, 1)$.
   
c. Use $L(x, y)$ from part b to approximate $0.02 \cos(0.98 \pi)$. 
13. (15 pts) Let $f(x, y) = 2xy + 4\sqrt{1+y}$.
   a. Find the directional derivative of $f$ in the direction of \( \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \) at the point \((1, 3)\).
   b. Find the maximum rate of change of $f$ at the point \((1, 3)\).
   c. Find the direction in which the maximum rate of change of $f$ at the point \((1, 3)\) occurs.
14. (15 pts) Let \( f(x, y, z) = z - \cos x \sin y \).

   a. Calculate \( \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \) at \((x, y, z) = (0, \pi, 1)\).

   b. Write an equation for the tangent plane \( P \) of the level surface \( f(x, y, z) = 1 \) at \((x, y, z) = (0, \pi, 1)\).

   c. Write an equation for the line passing through the point \((0, \pi, 1)\) and normal to the tangent plane \( P \) described in part b, i.e., find the normal line.
15. (15 pts) Consider

\[ f(x, y) = 3x^2y - 3x^2 + y^3 - 3y^2 + 2 \]

a. Find all critical points of \( f \).
b. Find local maximum, minimum values, and saddle points of \( f \).
c. Find the absolute maximum and minimum values of \( f \) on the closed triangular region in the \( xy \)-plane with vertices \((0, 0), (1, 0), (1, 1)\).
16. (15 pts) Use **Lagrange multipliers** to find the dimensions of a rectangular box with largest volume if the sum of its length and girth (perimeter of a cross-section perpendicular to the length) is 48 in.

*No credit* will be given if Lagrange multipliers are not used.