MATH 250
First Midterm Exam
Feb 22, 2006

Name:_____
Student Number:_____
Instructor:_____
Section:_____

There are **6** multiple choice questions and **5** partial credit questions. **In order to obtain full credit for the partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work on a partial credit problem. THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.**

For multiple choice problems, write the letter of your choice in the space provided below.

Your Answer :	**Points awarded**
1. (5 pts) _____	Q. 7 (10 pts) _____
2. (5 pts) _____	Q. 8 (10 pts) _____
3. (5 pts) _____	Q. 9 (20 pts) _____
4. (5 pts) _____	Q. 10 (15 pts) _____
5. (5 pts) _____	Q. 11 (15 pts) _____
6. (5 pts) _____	Total _____

1. (5 points) The solution of the initial value problem

$$\frac{dy}{dx} = (x^2 + e^x)y + e^{x^2}, \qquad y(0) = y_0$$

is defined for

(a) $x < 0$, because it is a non-linear equation.

(b) all real x, because it is a linear equation.

(c) all real x, because it is a non-linear equation.

(d) The equation has no solutions.

2. (5 points) The interval, where the solution to the initial value problem

$$(t-3)\frac{dy}{dt} - 3\frac{y}{t+1} = \frac{1}{t}, \; y(2) = -0.5,$$

exists and is unique, is

(a) $0 < t < \infty$,

(b) $0 < t < 3$,

(c) $-1 < t < 0$,

(d) $-1 < t < 3$.

3. (5 points) Which of the following functions is a solution of the following differential equation:

$$\frac{dy}{dt} = \frac{2t}{e^y}$$

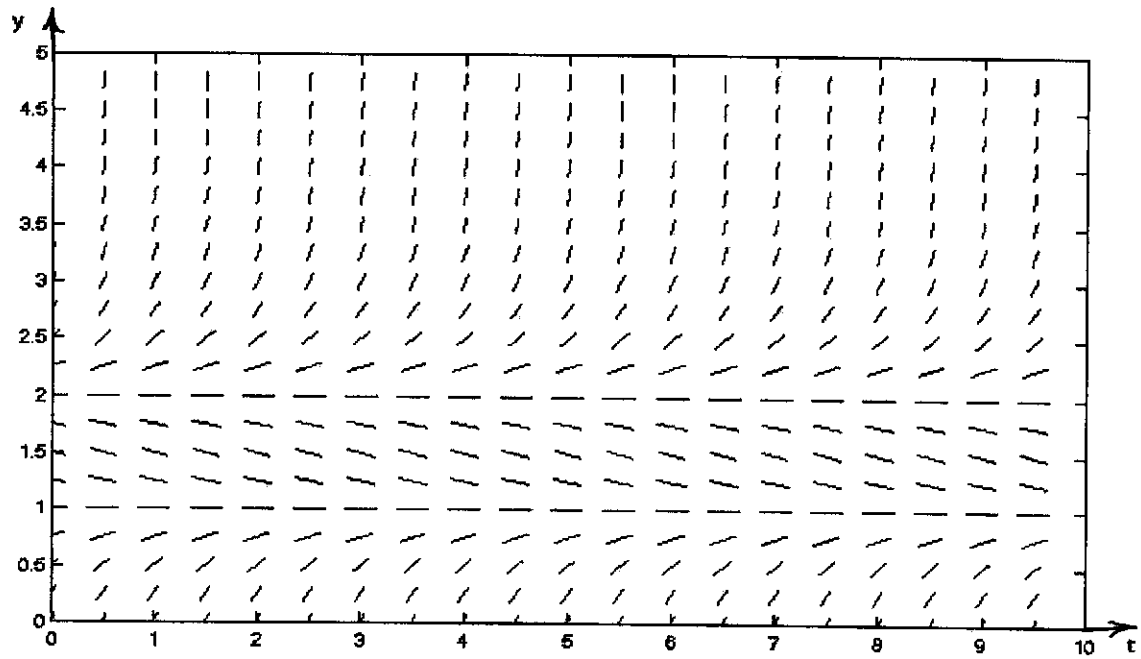
(a) $y(t) = 2e^t$

(b) $y(t) = 2\ln t$

(c) $y(t) = -2t^2$

(d) $y(t) = \frac{1}{t}$

4. (5 points) The following picture shows the direction field of an autonomous equation $y' = f(y)$ with two equilibrium solutions $y_1 = 1$ and $y_2 = 2$. Classify these equilibrium solutions.



(a) Both y_1 and y_2 are asymptotically stable.

(b) Both y_1 and y_2 are unstable.

(c) y_1 is asymptotically stable and y_2 is unstable.

(d) y_1 is unstable and y_2 is asymptotically stable.

5. (5 points) For which value of λ are the functions $f_1(t) = t^2 + t$ and $f_2(t) = t^2 + (1+\lambda)t + \lambda$ linearly dependent?

   (a) There is no such λ

   (b) $\lambda = 0$

   (c) $\lambda = 1/2$

   (d) For all λ

6. (5 points) Let $y_1 = (1+t)e^{2t}$ and $y_2 = e^{2t}$. Then their Wronskian $W(t)$ is

   (a) e^{2t}

   (b) 0

   (c) e^{-2t}

   (d) e^{4t}

7. (10 points) Solve the initial value problem

$$t^3\frac{dy}{dt} + t^2y = 2t^2 - 1, \qquad y(1) = 3.$$

8. (10 points)

Solve the initial value problem

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0, \qquad y(0) = 0, \quad y'(0) = 1.$$

9. (20 points) A tank originally contains 100 gallons of water with 20 lb of salt in solution. Water containing 0.1 lb of salt per gallon enters the tank at the rate of 5 gallons per minute, and a well-mixed solution leaves the tank at the same rate. Let $Q(t)$ be the amount of salt in the tank at time t.

   (a) (5 pts) Set up an initial value problem for $Q(t)$.

   (b) (10 pts) Find an expression $Q(t)$ by solving the initial value problem above.

   (c) (5 pts) Find the limit ρ_0 of concentration of salt as $t \longrightarrow \infty$.

10. (15 points) Consider the differential equation

$$\frac{dy}{dt} = f(y) = y^2(y^2 - 1).$$

(a) (5 pts) Find the equilibrium solutions of this differential equation. Sketch $f(y)$ and determine its sign as a function of y.

(b) (5 pts) Sketch the direction field of this equation and the graphs of a few representative solutions for $t \geq 0$.

(c) (5 pts) Classify the equilibrium solutions (asymptotically stable, unstable, or semi-stable).

11. (15 points) Find the solution of the initial value problem

$$(y+1)\frac{dy}{dt} = t^2 + 1,\ y(1) = -2.$$

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