Name: ______________________

MATH 250

Student Number: ______________________

Midterm Exam II

Instructor: ______________________

November 9, 2004

Section: ______________________

This exam has 10 questions for a total of 100 points. There are **5** partial credit questions. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.**

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION. At the end of the examination, the booklet will be collected.

For instructor use only:

Problem number	1	2	3	4	5	6	7	8	9	10	Total
Points	5	5	5	5	5	18	15	12	18	12	100
Your points											

1. (5 points) Which of the following is the general solution of $4y'' + y = 0$?

   (a) $y(t) = c_1 e^t + c_2 e^{-t}$,

   (b) $y(t) = c_1 \cos(t/2) + c_2 \sin(t/2)$,

   (c) $y(t) = (c_1 + c_2 t)e^{t/2}$,

   (d) $y(t) = e^{-t}(c_1 \cos(t) + c_2 \sin(t))$.

2. (5 points) Which of the following could be a particular solution to

$$y'' + 4y' + 4y = te^t + \cos(2t) + t + 2,$$

   for some constants $A_1, A_2, A_3, A_4, A_5, A_6$?

   (a) $y(t) = (A_1 + A_2 t)te^t + A_3 \sin(2t) + A_4 \cos(2t) + A_5 t + A_6$,

   (b) $y(t) = (A_1 + A_2 t)e^t + A_3 \sin(t) + A_4 \cos(t) + A_5 t + A_6$,

   (c) $y(t) = (A_1 + A_2 t)e^t + A_3 t \sin(2t) + A_4 t \cos(2t) + A_5 t + A_6$,

   (d) $y(t) = (A_1 + A_2 t)e^t + A_3 \sin(2t) + A_4 \cos(2t) + A_5 t + A_6$.

3. (5 points) Given the initial value problem:

$$t(t-2)y'' + \ln(t)y' + ty = 4(t-3), \quad y(3) = 1, \quad y'(3) = -1.$$

Which of the following is the largest interval on which we are guaranteed the existence of a unique solution to the initial value problem given above?

(a) $(2, \infty)$,

(b) $(-\infty, 0)$,

(c) $(0, 2)$,

(d) $(0, 3)$.

4. (5 points) The motion of a certain spring-mass system is governed by the following equation:

$$\frac{1}{2}u'' + 18u = 0.$$

What is the period of this vibration?

(a) $\frac{3}{\pi}$,

(b) $\frac{\pi}{6}$,

(c) 6,

(d) $\frac{\pi}{3}$.

5. (5 points) Which of the following pairs of functions are linearly independent?

   (a) $f(t) = t^3, \quad g(t) = 2t^3,$

   (b) $f(t) = \sin^2(t), \quad g(t) = \cos(2t) - 1,$

   (c) $f(t) = e^{t+2}, \quad g(t) = e^t,$

   (d) $f(t) = \sin(t), \quad g(t) = \cos(t).$

6. (18 points) A spring is stretched 0.1 meter by a mass that weights 10 kg. Neglect damping, and use $g = 10m/s^2$.

   (a) (10 points) Derive a differential equation that describe the motion of the mass, and find the general solution of this equation.

   (b) (5 points) If the mass is initially stretched 0.1 meter downward from equilibrium and then released from rest, find the position of the mass at any time $t > 0$.

   (c) (3 points) What is the frequency, period and amplitude of this motion?

7. (15 points) By using the method of undetermined coefficients, find the general solution of

$$y'' + 3y' + 2y = 3 + 2t.$$

8. (12 points) Solve the following initial value problem:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0, \qquad y(0) = 1, \quad y'(0) = -1.$$

9. (18 points) Given the differential equation

$$t^2y'' - ty' + y = 0.$$

(a) (6 points) Use Abel's theorem to find the Wronskian of two linearly independent solutions to this O.D.E. without solving it.

(b) (12 points) Given that $y_1(t) = t$ is a solution of this equation, use the Wronskian computed above, or some method of reduction of order, to compute a second solution y_2.

10. (12 points) Solve the following initial value problem:

$$y'' + 2y' + y = 0, \qquad y(0) = 1, \quad y'(0) = 0.$$