SPRING 05 FINAL EXAMINATION
MATH 140

1. (5 pts.) Evaluate $\lim_{h\to 0} \dfrac{\frac{1}{5+h} - \frac{1}{5}}{h}$.

   a) $-\dfrac{1}{25}$

   b) $\dfrac{1}{25}$

   c) $\dfrac{1}{5}$

   d) 5

   e) The limit does not exist.

2. (5 pts.) Evaluate $\lim_{x\to 16} \dfrac{4-\sqrt{x}}{16-x}$.

   a) $\dfrac{1}{16}$

   b) $\dfrac{1}{4}$

   c) 4

   d) 8

   e) $\dfrac{1}{8}$

3. (5 pts.) Find an equation of the tangent line to $y = x^2 + 7x$ at the point $(2, 18)$.

   a) $y = 11x - 4$

   b) $y = 11x + 4$

   c) $y = -13x - 4$

   d) $y = 11x - 6$

   e) $y = 12x + 4$

4. (5 pts.) The limit below represents the derivative of some function $f(x)$ at some number a. State $f(x)$ and a.

$$\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - 1}{h}$$

   a) $f(x) = \cos(x),\ a = \dfrac{\pi}{2}$

   b) $f(x) = \cos\left(x + \dfrac{\pi}{2}\right),\ a = 0$

   c) $f(x) = \sin\left(x + \dfrac{\pi}{2}\right),\ a = \dfrac{\pi}{2}$

   d) $f(x) = \sin(x),\ a = \dfrac{\pi}{2}$

   e) $f(x) = \sin(x),\ a = 1$

5. (5 pts.) Differentiate $h(x) = \dfrac{x+2}{x-8}$.

   a) $h'(x) = -\dfrac{10}{(x-8)^2}$

   b) $h'(x) = \dfrac{10}{(x-8)^2}$

   c) $h'(x) = -\dfrac{2}{(x-8)^2}$

   d) $h'(x) = \dfrac{2}{(x-8)^2}$

   e) $h'(x) = 1$

6. (5 pts.) Differentiate $g(x) = 2\sec x + \tan x$.

   a) $g'(x) = 2\sec x \tan x + 1 - \tan^2 x$

   b) $g'(x) = 2\sec x \tan x + \sec^2 x$

   c) $g'(x) = 2\sec x \tan x + 1 + \tan x$

   d) $g'(x) = 2\sec x \tan x + 1 - \sec x$

   e) $g'(x) = 2\tan^2 x + \sec x \tan x$

7. (5 pts.) Find the derivative of $y(x) = b^3 + 10\cos^3 x$ where b is a constant.

   a) $3b^2 - 30\cos^2(x)\sin(x)$

   b) $3b^2 - 30\cos^2(x)$

   c) $-30\cos^2(x)\sin(x)$

   d) $30\cos^2(x)$

   e) $-30\cos^2(x)$

8. (5 pts.) Find y' by implicit differentiation given $4\cos(x)\sin(y) = 7$.

   a) $y' = \tan(x)$

   b) $y' = \cot(x)\cot(y)$

   c) $y' = \tan(x)\tan(y)$

   d) $y' = \tan(xy)$

   e) $y' = 7$

9. (5 pts.) Find y'' given $y = \sqrt{6x+7}$.

   a) $y''(x) = -81(6x+7)^{-\frac{3}{2}}$

   b) $y''(x) = -9(6x+7)^{-\frac{3}{2}}$

   c) $y''(x) = \dfrac{3}{8}(6x+7)^{-\frac{3}{2}}$

   d) $y''(x) = -\dfrac{3}{8}(6x+7)^{-\frac{5}{2}}$

   e) $y''(x) = 9(6x+7)^{\frac{3}{2}}$

10. (5 pts.) Find the critical numbers of the function $y = \dfrac{x}{x^2+49}$.

a) $7, -7$

b) $49, -49$

c) $7, 0$

d) $0, -7$

e) $0, -49$

11. (5 pts.) Find the absolute maximum value of $y = \sqrt{81 - x^2}$ on the interval $[-9, 9]$.

a) 10

b) 9

c) 0

d) 8

e) 81

12. (5 pts.) Find the interval(s) on which the function $f(x) = x^3 - 108x + 10$ is increasing.

a) $(-\infty, -18),\ (18, \infty)$

b) $(-\infty, 6)$

c) $(-6, 6)$

d) $(-6, \infty)$

e) $(-\infty, -6),\ (6, \infty)$

13. (5 pts.) Find $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2+6x}}{6x+1}$.

a) $\dfrac{1}{5}$

b) $\dfrac{5}{6}$

c) 1

d) ∞

e) $\dfrac{1}{6}$

14. (5 pts.) Find $f(x)$ if $f'(x) = 3\cos(x) + 10\sin(x)$ and $f(0) = 3$.

a) $f(x) = 3\sin(x) - 10\cos(x) + 13$

b) $f(x) = 3\sin(x) + 10\cos(x) + 13$

c) $f(x) = -3\sin(x) - 10\cos(x) + 3$

d) $f(x) = -3\sin(x) - 10\cos(x)$

e) $f(x) = 3$

15. (5 pts.) Find the derivative of the function $g(x) = \displaystyle\int_4^{x^2} 3\sqrt{1+t^7}\, dt$.

a) $g'(x) = 3\sqrt{1+x^{14}}$

b) $g'(x) = 6x\sqrt{1+x^{14}}$

c) $g'(x) = 6x\sqrt{1+x^7}$

d) $g'(x) = 3\sqrt{1+x^7}$

e) $g'(x) = 3x^2\sqrt{1+x^{14}}$

16. (5 pts.) Evaluate the integral $\displaystyle\int_1^4 \frac{x^2+6}{\sqrt{x}}\, dx$.

a) 64

b) 32

c) 16

d) $\dfrac{122}{5}$

e) $\dfrac{32}{5}$

17. (5 pts.) Evaluate the indefinite integral $\displaystyle\int t^2 \cos(8 - t^3) dt$.

a) $\dfrac{1}{3}\sin(8-t^3) + C$

b) $\dfrac{1}{3}\sin(8-t^3)$

c) $\dfrac{1}{3}\cos(8-t^3) + C$

d) $-\dfrac{1}{2}\sin(8-t^3) + C$

e) $-\dfrac{1}{3}\sin(8-t^3) + C$

18. (5 pts.) Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \dfrac{1}{x}$ from $x = 7$ to $x = 9$.

a) 16π

b) $\dfrac{16\pi}{63}$

c) $\dfrac{2\pi}{63}$

d) $\dfrac{63\pi}{2}$

e) $\dfrac{7}{16\pi}$

19. (5 pts.) Find (by Cylindrical Shell Method) the volume of the solid obtained by rotating the region bounded by the curves $x = 6 + y^2$, $x = 0$, $y = 1$, $y = 3$ about the x-axis.

a) $V = 86\pi$

b) $V = 88\pi$

c) $V = 176\pi$

d) $V = 93\pi$

e) $V = 6\pi$

20. (5 pts.) Find a number c in $[0, 3]$ at which the value of the function $f(v) = 8 - v^2$ is equal to the average value of the function on the interval $[0, 3]$.

a) $c = \sqrt{2}$

b) $c = -\sqrt{3}$

c) $c = \dfrac{\sqrt{3}}{2}$

d) $c = \sqrt{3}$

e) $c = \sqrt{8}$

21. (10 pts.) A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\dfrac{\pi}{3}$ rad?

22. (10 pts.) A rectangular poster is to have the total area of 200 in^2 with 1-inch margins at the sides and 2-inch margins at the bottom and top. What dimensions will give the largest printed area?

23. (15 pts.) A region in the xy-plane is enclosed by the curves $y = x^2$ and $y = 8 - x^2$.

(a) (5 pts.) Draw the region. Clearly label the curves and intersection points.

(b) (5 pts.) Set up a formula for the area of this region.

(c) (5 pts.) Calculate this area.

24. (15 pts.) A region R in the xy-plane is bounded by $y = x^2$ and $y^2 = x$.

(a) (5 pts.) Sketch the region R. Be sure to label the curves and intersection points.

(b) (5 pts.) Set up the integral (by using the **Washer Method**) which measures the volume of the solid generated by revolving R around the x-axis. DO NOT evaluate the integral.

(c) (5 pts.) Set up the integral (by using the **Cylindrical Shell Method**) which measures the volume of the solid generated by revolving R around the x-axis. DO NOT evaluate the integral.