MATH 140 NAME _____

FINAL EXAMINATION STUDENT NUMBER _____

DECEMBER 16, 2004 INSTRUCTOR _____

FORM A SECTION NUMBER _____

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your scantron. On your scantron identify your name, this course (Math 140) and the date. Code and blacken the corresponding circles on your scantron for your student I.D. number, section number and your **test form.**

This exam is worth a total of 150 points. There are 19 multiple choice questions worth a total of 114 points. For each problem **five** possible answers are given, only one of which is correct. **Circle** the correct answer in your exam booklet **and blacken** the corresponding space on the **scantron form**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test is the same as the problem number of the scantron. There are **2** partial credit questions worth a total of 36 points. **In order to obtain full credit for problems 20 and 21, all work must be shown**. **Credit will not be given for an answer not supported by work**. The point value for each question is shown to the left of the question number.

ALL CALCULATORS, NOTES, BOOKS, ETC. ARE FORBIDDEN.

MC (114 pts.)	
20. (18 pts.)	
21. (18 pts.)	
Total	

Do not write in the box to the left.

6 pts 1. Find the $x-$coordinates of ALL the inflection points of the function $f(x)$ if

$$f''(x) \;=\; 7(x-4)^7(x-1)^2(x+1)(x+4)^6.$$

a) $x = 4$ and $x = 1$ only.

b) $x = 4$ and $x = -1$ only.

c) $x = -4$ and $x = 1$ only.

d) $x = -4$, $x = -1$ only.

e) $x = -4$, $x = -1$, $x = 1$, and $x = 4$ only.

6 pts 2. Assume that y is a differentiable function of x which satisfies the relation

$$x \sin y + y \cos x = \pi/2.$$

Find $\dfrac{dy}{dx}$ when $x = \pi$ and $y = \pi/2$.

a) $\dfrac{dy}{dx} = -1.$

b) $\dfrac{dy}{dx} = 1.$

c) $\dfrac{dy}{dx} = \dfrac{1}{2}.$

d) $\dfrac{dy}{dx} = -\dfrac{1}{2}.$

e) $\dfrac{dy}{dx} = 2.$

6 pts 3. It is given that $f'(x) = x^2 - 7x + 12$. On which interval(s) is $f(x)$ increasing?

a) $(3, 4)$ only.

b) $(-\infty, 3)$ and $(4, +\infty)$ only.

c) $(-\infty, 3)$ only.

d) $(4, +\infty)$ only.

e) $(0, +\infty)$ only.

6 pts 4. Find all of the vertical and horizontal asymptotes of the function

$$f(x) = \frac{2x - 1}{x^2 - 1}$$

a) $y = 2$ only.

b) $y = 2$ and $x = 1$ only.

c) $y = 2$, $x = 1$, and $x = -1$ only.

d) $y = 0$, $x = -1$, and $x = 1$ only.

e) $x = 0$, $x = 1$, and $x = -1$ only.

6 pts 5. Find the most general antiderivative of $f(x) = \dfrac{x^8 + x^4 + 1}{x^6}$ on the interval $(0, \infty)$.

a) $\dfrac{x^3}{3} - \dfrac{1}{x} - \dfrac{1}{7x^7} + C$

b) $\dfrac{x^3}{3} - \dfrac{1}{x} + \dfrac{1}{7x^7} + C$

c) $\dfrac{x^3}{3} - \dfrac{1}{x} - \dfrac{1}{5x^5} + C$

d) $\dfrac{7}{9}x^2 + \dfrac{7}{5x^2} + 7\dfrac{1}{x^6} + C$

e) $x - 2\dfrac{1}{x^3} - 6\dfrac{1}{x^7} + C$

6 pts 6. Find the equation of the tangent line to the curve $y = \tan\left(\dfrac{\pi x}{4}\right)$ at the point whose $x-$coordinate equals 1.

a) $y = 2x - 1$

b) $y = x + \tan\left(\dfrac{\pi x}{4}\right) + 1$

c) $y = 2x$

d) $y = x - 1$

e) $y = \left(\dfrac{\pi}{2}\right)x + \left(1 - \dfrac{\pi}{2}\right)$

6 pts 7. If $f(x) = \sqrt[3]{x^2+1}$, find $f''(0)$.

a) $f''(0) = \dfrac{10}{9}$

b) $f''(0) = -\dfrac{4}{9}$

c) $f''(0) = \dfrac{4}{9}$

d) $f''(0) = \dfrac{2}{3}$

e) $f''(0) = \dfrac{2}{9}$

6 pts 8. Find $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+2}}{3x-1}$

a) $\dfrac{2}{3}$

b) $-\dfrac{2}{3}$

c) $\dfrac{1}{3}$

d) $-\dfrac{1}{3}$

e) The limit does not exist.

6 pts 9. Evaluate $\int \frac{x}{\sqrt{3+2x}} dx$.

a) $3(x-3)\sqrt{3+2x}+C$

b) $\frac{2}{3}(x-6)\sqrt{3+2x}+C$

c) $\frac{1}{3}(x-3)\sqrt{3+2x}+C$

d) $\frac{2}{27}(3x-4)\sqrt{3+2x}+C$

e) None of the above.

6 pts 10. Find $\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)}$

a) 1

b) $\frac{5}{3}$

c) $+\infty$

d) 0

e) The limit does not exist.

6 pts 11. Let $f(x) = \begin{cases} \sqrt[3]{x^2+2x+C} & \text{if } x \leqslant 4 \\ 2|x|-5 & \text{if } x > 4 \end{cases}$. Find the value of C which makes f continuous at $x = 4$.

a) $C = 3$

b) $C = 27$

c) $C = 1$

d) $C = -27$

e) $C = -1$

6 pts 12. A water tank has a shape of an inverted cone with base radius $2m$ and height $3m$. If water is being pumped into the tank at a rate of $2m^3$ per hour, how fast is the water level rising when the height of the water in the tank is $h = 1$ meter? (Hint: The volume of a cone of height h and base radius r is $\frac{1}{3}\pi r^2 h$. In this case, $r = \frac{2}{3}h$.)

a) $\frac{9}{2\pi}$ m per hour.

b) $\frac{9\pi}{8}$ m per hour.

c) $\frac{4}{3\pi}$ m per hour.

d) $\frac{3\pi}{4}$ m per hour.

e) There isn't enough information to determine the answer.

6 pts 13. A bath was filled with hot water at 132 degrees Fahrenheit. Three minutes later, the temperature of the bath water had dropped to 99 degrees Fahrenheit. What allows you to conclude that

1) The bath water must have been at 110 degrees Fahrenheit at some time during these three minutes.

2) There was a time during these three minute when the bath water was cooling at the (instantaneous) rate of 11 degrees Fahrenheit per minute.

a) Mean Value Theorem for 1) and Intermediate Value Theorem for 2)

b) Mean Value Theorem for 1) and First Derivative Test for 2)

c) Intermediate Value Theorem for 1) and Mean Value Theorem for 2)

d) Intermediate Value Theorem for 1) First Derivative Test for 2)

e) First Derivative Test for 1) and Mean Value Theorem for 2)

6 pts 14. The sum of two positive numbers is 4. Find the smallest possible sum of the first number and the square of the second.

a) $\frac{7}{2}$

b) $\frac{1}{2}$

c) 1

d) 4

e) $\frac{15}{4}$

6 pts 15. Find $\int_0^{\pi} \cos\left(\frac{x}{2}\right)\, dx$

a) 0

b) 1

c) -1

d) 2

e) -2

6 pts 16. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x^3}$, $x = 1$, $x = 2$, and $y = 0$ about the x–axis.

a) 4

b) 16π

c) $\frac{15\pi}{4}$

d) $\frac{15}{4}$

e) 4π

6 pts 17. A water container is filled with water at a rate of $r(t)$ liters per minute. If the container had 3 liters of water at time $t = 0$, how much water does it contain at time $t = 10$?

a) $3 + \int_0^{10} r(t)dt$

b) $\int_3^{10} r(t)\,dt$

c) $r'(10) + 3$

d) $10r'(3)$

e) none of the above

6 pts 18. Evaluate $\int (x + x^{1/2})(1 + x^{3/2})dx$.

a) $\frac{2x^{3/2}}{3} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{2x^{7/2}}{7} + C$

b) $\frac{3x^{2/3}}{2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{7x^{2/7}}{2} + C$

c) $\frac{2x^{5/2}}{3} + \frac{4x^4}{15} + \frac{x^3}{3} + \frac{x^{9/2}}{5} + C$

d) $2x^{1/2} + \frac{x^2}{2} + \frac{x^3}{3} + \frac{2x^{5/2}}{5} + C$

e) None of the above.

6 pts 19. Which of the following can be interpreted as the volume of the solid generated by rotating the region bounded by $y = x^3$ and $y = x^2$ about the $x-$axis?

a) $\displaystyle\int_0^1 (x^3 - x^2)dx$

b) $\displaystyle\int_0^1 (x^2 - x^3)dx$

c) $\displaystyle\pi\int_0^1 (x^6 - x^4)dx$

d) $\displaystyle\pi\int_0^1 (x^4 - x^6)dx$

e) $\displaystyle\pi\int_0^1 (x^2 - x^3)^2dx$

18 pts 20. A region in the xy–plane is enclosed by the curves $y = x^2$, $y = 2x$, $x = 0$, and $x = 3$.

a) (2 points) Find the intersection points of the curves $y = x^2$ and $y = 2x$.

b) (4 points) Draw the region. Clearly label the four curves and shade the region.

c) (6 points) Set up a formula for the area of this region.

d) (6 points) Calculate this area.

18 pts 21. A region R in the $xy-$plane is bounded by $y = \sqrt{x}$, $x + y^2 = 2$ and $y = 0$.

a) (6 points) Sketch the region R. Be sure to label the curves and intersection points.

b) (6 points) Set up one integral which measures the volume of the solid generated by revolving R around the x-axis. DO NOT evaluate the integral.

c) (6 points) Set up one integral which measures the volume of the solid generated by revolving R around the y-axis. DO NOT evaluate the integral.