MATH 110

EXAMINATION II

NOVEMBER 18, 2002

NAME ____________________

STUDENT NUMBER ____________________

INSTRUCTOR ____________________

SECTION NUMBER ____________________

The examination consists of **20** multiple choice questions For this exam calculators are **not** allowed and are not needed. For each problem, please fill in the bubble on the scantron sheet and circle the correct answer on your examination. Each problem is worth five points.

THE USE OF CALCULATORS IS <u>NOT</u> PERMITTED IN THIS EXAMINATION.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE **20** PROBLEMS ON **11** PAGES (INCLUDING THIS ONE).

1. Let $f(x) = x^3 - 3x^2 + 4$. Find the intervals where f is increasing and intervals where f is decreasing.

   a) f is increasing on $(0, \infty)$; decreasing on $(-\infty, 0)$.

   b) f is increasing on $(-\infty, 0)$ and on $(2, \infty)$; decreasing on $(0, 2)$.

   c) f is increasing on $(0, 2)$; decreasing on $(-\infty, 0)$ and on $(2, \infty)$.

   d) f is increasing on $(-\infty, 2)$; decreasing on $(2, \infty)$.

2. Let $h(x) = \dfrac{x^2}{x^2 + 1}$. Find the relative maxima and relative minima of h.

   a) relative maximum at $x = 1$; relative minimum at $x = 0$.

   b) no relative maxima or minima.

   c) relative minimum at $x = 0$; no relative maximum.

   d) relative maximum at $x = 0$; no relative minimum.

3. Let $f(x) = xe^x$. Determine the intervals of concavity of f.

a) f is concave upward on $(-2, \infty)$; concave downward on $(-\infty, -2)$.

b) f is concave upward on $(0, \infty)$; concave downward on $(-\infty, 0)$.

c) f is concave upward on $(-1, \infty)$; concave downward on $(-\infty, -1)$.

d) f is concave upward everywhere.

4. Find the relative extrema (if any) of the function $f(x) = x^2 + \dfrac{2}{x}$.

a) there are no relative extrema.

b) a relative maximum at $x = 0$.

c) a relative maximum at $x = 1$.

d) a relative minimum at $x = 1$.

5. Determine all vertical and horizontal asymptotes of the function

$$h(x) = \frac{2x^2 - 3}{x^2 + x}.$$

a) vertical asymptote at $x = 0$ and $x = -1$; horizontal asymptote $y = 2$.

b) vertical asymptote at $x = 0$ and $x = -1$; no horizontal asymptote.

c) vertical asymptote at $x = -\sqrt{\frac{3}{2}}$ and $x = \sqrt{\frac{3}{2}}$; horizontal asymptote $y = 2$.

d) vertical asymptote at $x = -\sqrt{\frac{3}{2}}$ and $x = \sqrt{\frac{3}{2}}$; no horizontal asymptote.

6. A company's average cost (in dollars) for producing x units of their product is

$$\overline{C}(x) = 2.5 + \frac{1000}{x}.$$

What is the horizontal asymptote of $\overline{C}$, and what is the limiting value of the average cost?

a) $x = 0$ and \$0

b) $x = 0$ and \$1000

c) $y = 2.5$ and \$2.50

d) $y = 2.5$ and \$0

7. Find the absolute maxima and absolute minima (if any) of the function

$$f(x) = x - 2\sqrt{x} \quad \text{on} \quad [0, 9].$$

a) absolute maximum is 3; absolute minimum is 0.

b) absolute maximum is 0; absolute minimum is -1.

c) absolute maximum is 9; absolute minimum is 0.

d) absolute maximum is 3; absolute minimum is -1.

8. The daily profit function (in dollars) for producing x units of a certain product is

$$P(x) = \frac{-2x^3}{1,000,000} + 6x - 400.$$

What is the largest possible daily profit?

a) \$3000

b) \$3600

c) \$4000

d) \$7200

9. If $x + 2y = 10$, what is the minimum value of $x^2 + y^2$?

   a) 10

   b) 20

   c) 25

   d) 30

10. A cylindrical can with radius r and height h has volume $V = \pi r^2 h$ and total surface area $A = 2\pi(r^2 + rh)$. If the volume is 16π cubic inches, what values of r and h minimize the surface area?

   a) $r = \sqrt{2}$ in, $h = 4$ in.

   b) $r = 4$ in, $h = 1$ in.

   c) $r = 2$ in, $h = 4$ in.

   d) $r = 2$ in, $h = 3$ in.

11. If $3^{t+1} = \left(\frac{1}{9}\right)^{2t}$, what is t?

a) $t = \frac{1}{2}$

b) $t = -\frac{1}{2}$

c) $t = \frac{1}{4}$

d) $t = -\frac{1}{5}$

12. Given that $\ln 2 \approx 0.69$ and $\ln 3 \approx 1.10$, what is $\ln \sqrt[3]{2}$?

a) $\ln \sqrt[3]{2} \approx 1.79$

b) $\ln \sqrt[3]{2} \approx 0.23$

c) $\ln \sqrt[3]{2} \approx 0.41$

d) $\ln \sqrt[3]{2} \approx 0.55$

13. If \$1000 is invested at 7% per year compounded *continuously*, what will be the accumulated amount after 10 years?

a) $1000e^{0.7}$ dollars.

b) $1000(0.07)^{10}$ dollars.

c) $1000(1.07)^{10}$ dollars.

d) $1000(1+e^{0.07})^{10}$ dollars.

14. What amount must be invested now at 6% per year compounded *monthly* to yield \$10,000 in 10 years?

a) $10,000(1.06)^{-10}$ dollars

b) $10,000e^{-1.06}$ dollars

c) $10,000(1.005)^{-120}$ dollars

d) $10,000(1.006)^{-12}$ dollars

15. What is $\dfrac{d}{dx}\left[\ln(2x^3)\right]$?

a) $\dfrac{3}{x}$

b) $\dfrac{1}{2x^3}$

c) $\dfrac{\ln x}{2x^3}$

d) $6x^2 \ln(2x^3)$

16. Find $f'(x)$ if $f(x) = (x+1)e^{x^2}$.

a) $2xe^{x^2}$

b) $e^{x^2}(3x+1)$

c) $e^{x^2}(2x^2+2x+1)$

d) $e^{x^2}(x^2+2x+1)$

17. Find the maximum value of $f(x) = \dfrac{x}{e^x}$.

a) e

b) 1

c) $\dfrac{1}{e}$

d) there is no maximum.

18. Find $\dfrac{d^2}{dx^2}\left(x \ln x\right)$.

a) $\dfrac{1}{x}$

b) $(x+1)\ln x$

c) $-\dfrac{1}{x^2}$

d) $1 + \dfrac{1}{x^2}$

19. Use logarithmic differentiation to find $f'(x)$ if $f(x) = \dfrac{x^5}{\sqrt{x^2+1}}$.

a) $\dfrac{x^5}{\sqrt{x^2+1}}\left(4x^4 - \dfrac{x}{\sqrt{x^2+1}}\right)$

b) $\dfrac{x^5}{\sqrt{x^2+1}}\left(\dfrac{5}{x} - \dfrac{x}{x^2+1}\right)$

c) $\dfrac{4x^4}{x^2+1}\left(\dfrac{5}{x} + \dfrac{1}{\sqrt{x^2+1}}\right)$

d) $\dfrac{4x^4\sqrt{x^2+1}+2x^6}{x^2+1}$

20. A new employee can produce 20 widgets per hour, and the employee's hourly production rate grows according to the learning curve

$$Q(t) = 40 - 20e^{-0.05t},$$

where t is the number of days on the job. How long does it take before a new employee can produce 30 widgets per hour?

a) $t = 0.05\ln\left(\dfrac{1}{2}\right)$ days

b) $t = \dfrac{20}{\ln 2}$ days

c) $t = \dfrac{\ln 2}{0.05}$ days

d) $t = 20\ln\left(\dfrac{1}{2}\right)$ days

21. KEY: 1-b, 2-c, 3-a, 4-d, 5-a, 6-c, 7-d, 8-b, 9-b, 10-c, 11-d, 12-b, 13-a, 14-c, 15-a, 16-c, 17-c, 18-a, 19-b, 20-c.