4.1 Polynomial Functions and their Graphs

- End behavior:
  - Look at leading coefficient/exponent and check sign
  - If polynomial is factored, check sign of each factor and multiply
- Graphing a polynomial:
  - Factor
  - Find x- and y-intercepts
  - Find end behavior
  - Either use test points between the intercepts or memorize the shape around zeros depending on the multiplicity:
    - If multiplicity is 1, then it crosses the x-axis in a straight line
    - If multiplicity is even, then it turns back around
    - If multiplicity is odd > 1, then it "squiggles" through the x-axis

4.2 Dividing Polynomials

- Long Division: Make sure to fill in missing powers
- Synthetic Division: Only works for division by $(x - c)$. Again make sure to fill in 0's for missing powers
- Remainder Theorem: to find $P(c)$ carry out a synthetic division for c, the remainder is $P(c)$
- Factor Theorem: c is a zero of $P \leftrightarrow (x - c)$ is a factor of $P(x)$

4.3 Real Zeros of Polynomials

- Rational Zeros Theorem: The possible rational zeros of a polynomial are of the form $\frac{p}{q}$ where p is a factor of the constant coefficient a_0 and q is a factor of the leading coefficient a_n
- How to find all zeros of a polynomial:
  - Try previous factoring methods like substitution or grouping, if this does not work then:
  - List all possible rational zeros using the Rational Zeros Theorem
  - Test the possible zeros
  - If you find a zero, factor it out
  - Repeat from the top until your polynomial is quadratic, then factor/complete the square/quadratic formula

4.4 Complex Zeros and the Fundamental Theorem of Algebra

- Fundamental Theorem of Algebra: every polynomial of degree n has precisely n zeros (zeros of multiplicity k are counted k times)
- Conjugate Zeros Theorem: If a complex number is a zero of polynomial with real coefficient, then its conjugate is also a zero

4.5 Rational Functions

- Horizontal asymptotes: n is the degree of the numerator, m is the degree of the denominator
  - $n > m$: no horizontal asymptote
  - $n = m$: horizontal asymptote is $y = \frac{a_n}{b_m}$
  - $n < m$: horizontal asymptote is $y = 0$
- Vertical asymptotes: zeros of the denominator (that do not cancel with the numerator)
- Graphing rational functions:
  - Factor numerator and denominator
  - Find x- and y-intercepts
  - Find horizontal and vertical asymptotes
  - Either use test points between intercepts/vertical asymptotes or use the shape around vertical asymptotes/intercepts to determine the shape of the graph
- Slant asymptote: only exists if the degree of the numerator is one greater than the degree of the denominator: use long/synthetic division

5.1 Exponential Functions

- $f(x) = a^x$, memorize the graph:
  - Horizontal asymptote $y = 0$
  - no vertical asymptote
  - Domain $= (-\infty, \infty)$
  - Range $= (0, \infty)$
- Compound interest formula: $A(t) + P(1 + \frac{r}{n})^{nt}$
- Continuously compounded interest: $A(t) = e^{rt}$

5.2 Logarithmic Functions

- Definition of logarithm: $\log_b a = x \leftrightarrow b^x = a$
- Properties:
  - $\log_b 1 = 0$
  - $\log_b b = 0$
  - $\log_b b^x = x$
  - $b^{\log_b x} = x$
- $f(x) = \log_b x$, memorize the graph:
  - Vertical asymptote: $x = 0$
  - no horizontal asymptote
  - Domain $= (0, \infty)$
  - Range $= (-\infty, \infty)$
- Finding the domain of logarithmic function: logarithms only defined for positive numbers
- Common log: $\log x = \log_1 0x$
- Natural log: $\ln x = \log_e x$

5.3 Laws of Logarithms

- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b \frac{x}{y} = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- no laws for $\log_b(x + y)$ or $\log_b x \cdot \log_b y$
- Change of base: $\log_b x = \frac{\log_c x}{\log_c b}$ where c can be any positive number

5.4 Exponential and Logarithmic Equations

- Solving exponential equations:
  - Isolate the exponential term on one side
  - Take logarithm of both sides:
    - If there is only one exponential term, use that base for the log
    - If there is an exponential term on both sides, use either the common or natural log
  - Pull the exponent to the front and solve the equation
- Solving logarithmic equations:
  - If there are multiple logarithmic terms, combine them into one using logarithmic laws
  - Isolate the logarithmic term on one side
  - Raise the base of the logarithm to the left and the right side of the equation
  - Use the property $b^{\log_b x} = x$ to get rid of the log
  - Solve the equation
- Two special cases of exponential equations:
  - Combination of exponential and polynomial terms: try to factor
  - Sum of multiple exponential terms: try to use substitution

5.5 Modeling with Exponential and Logarithmic Functions

- Exponential growth model: $n(t) = n_0 e^{rt}$
- To solve any problem you usually have to find n_0 and r
- Formulas and logarithmic scales

6.1 Angle Measure

- Relationship between Degrees and Radians:
  - convert from degrees to radians by multiplying by $\frac{\pi}{180}$
  - convert from radians to degrees by multiplying by $\frac{180}{\pi}$
- Coterminal angles: Angle between $0°$ and $360°$ degrees (or 0 and 2π)
- Length of a circular arc: $s = r\theta$ (θ in rad)
- Area of a circular sector: $A = \frac{1}{2}r^2\theta$ (θ in rad)
- Linear Speed and Angular Speed: $\omega = \frac{\theta}{t}$ and $v = \frac{s}{t}$
- Relationship between linear and angular speed: $v = r\omega$

6.2 Trigonometry of Right Triangles

- Trigonometric Ratios:
  - $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
  - $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
  - $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$
  - $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$
  - $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
  - $\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$
- Values of the trig ratios for angles $30°$, $45°$ and $60°$
- Solving right triangles

6.3 Trigonometric Functions of Angles

- Memorize in which quadrants each trig function is positive
- Reference angles: Acute angle formed by x-axis and terminal side
- Using reference angles to evaluate trig functions
- Reciprocal Identities:
  - $\csc\theta = \frac{1}{\sin\theta}$
  - $\sec\theta = \frac{1}{\cos\theta}$
  - $\cot\theta = \frac{1}{\tan\theta}$
  - $\tan\theta = \frac{\sin\theta}{\cos\theta}$
  - $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- Pythagorean Identities:
  - $\sin^2\theta + \cos^2\theta = 1$
  - $\tan^2\theta + 1 = \sec^2\theta$
  - $\cot^2\theta + 1 = \csc^2\theta$
- Expressing trig functions in terms of other trig functions
- Evaluating trig functions using identities
- Area of a Triangle: $\frac{1}{2}ab\sin\theta$ (where θ is the angle between a and b)

6.4 Law of Sines

- Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Solving triangles:
  - SAA
  - SSA (either no solution, one solution or two solutions)

6.5 Law of Cosines

- Law of Cosines:
  - $a^2 = b^2 + c^2 - 2bc\cos A$
  - $b^2 = a^2 + c^2 - 2ac\cos B$
  - $c^2 = a^2 + b^2 - 2ab\cos C$

  Solving triangles:
  - SSS
  - SAS
- Navigation: Bearing
- Heron's Formula: Area of a triangle is $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$