

CSE/MATH Homework IV

Due Nov 13, 2006

1. 6.4, pp 306.

2. 6.9, pp 306.

3. Consider a pendulum system consisting of a point mass m and a massless spring with spring constant $\frac{1}{\epsilon^2}$. $x = (x_1, x_2)$ and (mv_1, mv_2) are used as the coordinate and momenta for the mass respectively. The total energy of the system is given by,

$$H = K + U = \frac{1}{2}(mv_1^2 + mv_2^2) + mgx_2 + \frac{1}{2\epsilon^2}(\sqrt{x_1^2 + x_2^2} - l)^2,$$

where l is the natural length of the spring and g denotes the gravity. The equation of motion for the mass is given by,

$$mx'' = -\nabla U.$$

Choose the parameters $m = 1, l = 1, g = 1$, and $\epsilon = 10^{-5}$. Integrate the system using the three-stage Runge-Kutta-Gauss method with stepsize $\Delta t = 0.01$ until $T = 20$. Initially the mass is held at x_0 . Choose $x_0 = (l, 0)$ and $x_0 = (l + \epsilon, 0)$. For the numerical output, plot the position (both components), velocity (both components), and total energy.

4. The system,

$$\begin{cases} x_1' &= -K_1x_1x_2 + K_2x_3, \\ x_2' &= -K_1x_1x_2 + K_2x_3 - K_3x_2x_3, \\ x_3' &= K_1x_1x_2 - K_2x_3 - K_3x_2x_3, \end{cases}$$

describes a chemical process. x_1 and x_2 represent the concentration of the species A and B, and x_3 represents the concentration of an intermediate species X. Solve these differential equations using the two-stage and three-stage Runge-Kutta-Radau method. For the computer simulation, pick the parameters $K_1 = K_2 = 0.1, K_3 = 100$, the step size $\Delta t = 0.05, T = 100$, initial condition $(1, 1, 0)$. Compare the results to an explicit method.