CSE/MATH Homework III

Due Oct 30, 2006

1. Study the local consistency error of the numerical scheme,

$$x_{j+1} = -\frac{3}{2}x_j + 3x_{j-1} - \frac{1}{2}x_{j-2} + 3\Delta t f(t_j, x_j).$$

Is the method stable?

2. 7.3, page 384.

3. Consider the differential equation,

$$x' = 3x, x(0) = 1.$$

Solve the equation for $t < 1$ using the second order Adams-Bashforth method with the starting value generated from

i. the Euler's method,

ii. the second order Runge-Kutta method.

Use various step size and tabulate the numerical error by comparing the numerical solution with the exact solution. From the numerical error, conclude the order of accuracy of these two algorithm.

4. The system,

$$\begin{cases} x_1' &= -K_1 x_1 x_2 + K_2 x_3, \\ x_2' &= -K_1 x_1 x_2 + K_2 x_3 - K_3 x_2 x_3, \\ x_3' &= K_1 x_1 x_2 - K_2 x_3 - K_3 x_2 x_3, \end{cases}$$

describes a chemical process. x_1 and x_2 represent the concentration of the species A and B, and x_3 represents the concentration of an intermediate species X. Solve these differential equations using the second order and third order Adams-Bashforth method. For the computer simulation, pick the parameters $K_1 = K_2 = 1$, $K_3 = 10$, the step size $\Delta t = 0.01$, $T = 10$, initial condition $(1, 1, 0)$.