

CSE/MATH Homework I

Due Oct 2, 2006

1. Exercise 4.1, page 183.

2. Consider the implicit (backward) Euler's method, for which the evolution operator satisfies,

$$\Psi^{t+\tau,t}x = x + \tau f(t + \tau, \Psi^{t+\tau,t}x).$$

a. Compute the consistency error.

b. Consider a hybrid method,

$$\Psi^{t+\tau,t}x = x + \tau f\left(t + \frac{1}{2}\tau, \frac{1}{2}x + \frac{1}{2}\Psi^{t+\tau,t}x\right),$$

and compute the consistency error.

3. The system,

$$\begin{cases} x_1' &= -x_2, \\ x_2' &= x_1, \end{cases}$$

describes an oscillator with constant amplitude. The trajectory remains on a circle. Choose initial condition $x(0) = (1, 0)$, and uniform mesh $\Delta t = 0.4$. Solve the equations with the Euler method and the 4th order Runge-Kutta method for the time period $[0, 8\pi]$. Plot the trajectories on the $x_1 - x_2$ plane and compare them with the unit circle. Refine the mesh to $\Delta t = 0.1$, and repeat the calculation.

4. Consider the system,

$$r'' = (e^{-(r-r_0)} - 1)e^{-(r-r_0)} - \xi r'.$$

a. For the initial condition $r(0) = 2$ and $r'(0) = 0$, solve the equation using the 4th order Runge-Kutta method with $\Delta t = 0.01$ and $T = 200$. The parameters $r_0 = 1, \xi = 2$. Plot r'^2 in time.

b. Allow ξ to change in time, and couple the above equation with,

$$\xi' = Q(r'^2 - T),$$

where $Q = 0.1$ and $T = 0.5$. Solve the extended system using the 4th order Runge-Kutta method with the same step size. Plot r'^2 in time.

c. Vary the parameters Q and T and repeat the computation.